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MOTION OF AN ELASTOVISCOUS LIQUID WITHIN A TUBE AFTER REMOVAL OF A PRESSURE DIFFERENTIAL

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The nonisothermal flow of a nonlinear hereditary liquid within a ring-shaped channel after instantaneous removal of a pressure differential is studied.

The present study is a continuation of [1], which considered flow development after impulsive application of a pressure differential. We will now consider halting of a flow upon instantaneous removal of a pressure differential. Analytical solutions of this problem for the linear formulation have been presented in [2, 3]. In [4] a numerical calculation of flow in a circular tube for a two-constant Oldroyd model was performed. However, until the present there has been no study of the effect on rheodynamics of such important factors as nonisothermal conditions, the relaxation time spectrum, and the dependence of that spectrum and the relaxation moduli on shear velocity.

As in [1], we will employ a nonlinear integral rheological equation of state (RES):

$$\mathbf{T}=\int\limits_{-\infty}^{t} m[t-t',\ S_{\mathbf{D}}(t')]\left[\left(1+\frac{\varepsilon}{2}\right)(\mathbf{C}_t^{-1}(t')-\mathbf{E})+\frac{\varepsilon}{2}(\mathbf{C}_t(t')-\mathbf{E})\right]dt',$$
$$m=\sum_{k=1}^{\infty}\frac{\eta_k}{\lambda_k}f_k(S_{\mathbf{D}}(t'))\exp\left[-\int\limits_{t'}^{t}\frac{g_k(S_{\mathbf{D}}(t''))}{\lambda_k}dt''\right], \tag{1}$$
$$S_{\mathbf{D}}^2=2\,\mathrm{tr}\,\mathbf{D};\ \eta_k=\eta_0/\zeta(\alpha)k^{\alpha};\ \lambda_k=\lambda/k^{\alpha},\ 1.5\lesssim\alpha\lesssim 8.$$

Calculations were performed for a liquid the properties of which are independent of the deformation rate $f_h = g_h = 1$, and for three nonlinear models — the Bird–Carro (BC), Meister (M), and Macdonald–Bird–Carro (MBC) (see [1, 5]). The problem is formulated mathematically for a tube, the length of which significantly exceeds the extent of the hydrodynamic and thermal input segments:

$$\rho\frac{\partial v_z}{\partial t}=-\frac{\partial p}{\partial z}(t)+\frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}),\ R_1\leqslant r\leqslant R_2, \tag{2}$$

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$$v_z|_{r=R_1} = v_z|_{r=R_2} = 0, \quad \frac{\partial p}{\partial z} = \begin{cases} \text{const}, & 0 \leqslant t < t^*, \\ 0, & t \geqslant t^*. \end{cases}$$

If the interior and exterior channel surfaces are maintained at constant but differing temperatures θ_2 and θ_1, and dissipative heat liberation is low, then the well-known logarithmic temperature distribution will be established across the gap:

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{\ln(r/R_1)}{\ln(R_2/R_1)}. \quad (3)$$

The nonisothermal flow is described with the aid of the temperature-time invariance principle [6]. Neglecting the temperature-density correction, we may write $\eta=\eta_s a(\theta)$, $\lambda=\lambda_s a(\theta)$, while $a(\theta) = \exp\{(E_a/R)(1/\theta-1/\theta_s)\}$. The subscript s refers to the characteristic arithmetic mean temperature $\theta_s=0.5(\theta_2+\theta_1)$. The magnitude and sign of the temperature head are determined by the parameter $\nu=(\theta_1-\theta_2)/\theta_s$, while the temperature dependence of the viscosity is determined by $b = R\theta_s/E_a$.

Differentiation with respect to time reduces Eq. (1) to an equivalent system of differential equations (see [1]). This system is solved numerically using an implicit conservative finite-difference technique.

The parameters of the problem are the following: $\mathrm{El}=\lambda\eta_0/\rho h^2$, elasticity number; α, spectral characteristic; $\mathrm{We} = \lambda V/h$, Weissenberg number; $\delta=h/R_1 \equiv (R_2-R_1)/R_1$, relative gap. The ranges of α, ν, El, We, b are the same as in [1].

After removal of a pressure differential, reverse flows develop in the elastoviscous liquid. In the same situation an inelastic liquid is braked smoothly, continuing to move in the same direction. The presence of elastic properties leads to velocity and tangent stress oscillations during halting of the flow. For liquids with constant properties, due to the linearity of Eqs. (1), (2), for $f_k = g_k = 1$ for $t > t^*$ we have $T_{rz}(r, t) = T^{H}_{rz}(r, t) - T^{H}_{rz}(r, t-t^*)$, $v_z(r, t) = v^{H}_z(r, t) - v^{H}_z(r, t-t^*)$. Here T^{H}_{rz}, v^{H}_z is the solution of the problem of flow development for impulsive application of a pressure differential. The volume of liquid which passes through the tube Z(t) over a time $t > t^*$ will be given by

$$Z(t) = Z'(t) - Z'(t-t^*) = \int_0^{t^*} Q(t-t^*+t')\,dt'.$$

Here Q(t) is the volume flow rate and Z'(t) is the liquid volume for pulsed application of a pressure gradient. As $t \to \infty$ the flow rate tends to a stationary value

$$Q_{st} = -\frac{\pi}{8\eta_0}\left(\frac{\partial p}{\partial z}\right)\left[R_2^4 - R_1^4 - \frac{(R_2^2-R_1^2)^2}{\ln(R_2/R_1)}\right],$$

so that $Z(\infty) = Q_{st}t^*$. Thus, with both application and removal of a pressure differential, identical quantities of linear elastoviscous and Newtonian liquids flow though the channel.

The effect of the parameters α and El on flow characteristics is the same as in the case of pressure increase, increase in these parameters intensifying the manifestations of elastic properties. Figure 1 presents a diagram which relates the relative flow rate $\overline{Q}$ to the dimensionless stress on the exterior cylinder T_{rz_1}. Such diagrams were first proposed in [2]. For an elastoviscous liquid the values of $\overline{Q}$ and T_{rz_1} can significantly exceed the stationary values during development and halting of the flow. For large values of α the liquid oscillates more intensely. This is shown by the "curling" of the curve around the stationary point.

For nonlinear models the problems of pressure differential application and removal lose their symmetry. The values of the dimensionless pressure removal time $t^*\eta/\rho h^2$ will be

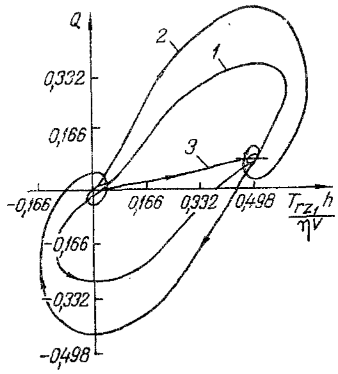

Fig. 1. $\bar{Q}$—T_{rz_1} diagram for elastoviscous (El = 10, δ = 0.1) at α = 2 (1) and α = 3 (2) and Newtonian liquids. Line denotes time of pressure removal and arrow indicates direction of time flow.

assumed fixed. Depending on the values of α, El, and We, the flow will or will not become developed by this time. The further the flow from the stationary state, the more intense the reverse flow after removal of $\partial p/\partial z$. For the elastoviscous liquid, the reversible deformation can be characterized conveniently by the ratio $Q_{max}(t > t^*)/Q(t^*)$, where $Q(t^*)$ is the flow at moment t^*; $Q_{max}(t > t^*)$ is the maximum flow rate of the counterflow after removal of the pressure gradient. The irreversible component is given by $Z(\infty)/Z_n(\infty)$ — the volume of liquid passed through the tube up to this value for a Newtonian liquid with viscosity η_0. After removal of the pressure gradient the tangent stresses across the gap reach their steady state values before halting of the flow, i.e., there exists a quasistationary flow stage which is realized at zero tangent stress.

For the BC model the $\bar{Q} - T_{rz_1}$ diagram, as in the case of a liquid with constant properties, is "curled" around a stationary value, However, deviation of the relative velocity from stationary values during oscillations in the BC model for given values of α and El are more intense than in the linear model, and the "twisting" is more severe. In Fig. 2 one can easily see the quasistationary regions, where stress on the wall is practically constant and flow rate increases (for application of pressure) or decreases to zero (for removal of pressure).

For the MBC model the flow rate values for application and removal of pressure go through two oscillations, with the first the larger. In the $\bar{Q} - T_{rz_1}$-diagram the second flow rate oscillation corresponds to formation of "loops" in the curve (Fig. 3). Pressure removal appears similar at low We in the M model.

Increase in relaxation time and spectrum parameter intensifies the manifestations of liquid elastic properties: reversible deformations increase. Thus, for the BC model, the ratio $Q_{max}(t > t^*)/Q(t^*)$ at We = 10 equals 0.89 for El = 10, α = 2; 1.49 for El = 10, α = 3; 3.57 for El = 100, α = 2, δ = 1. The elastic properties are also manifested more intensely for lower pressure gradients; at δ = 1, α = 2, El = 10, with change in We from 10 to 1 the ratio $Q_{max}(t > t^*)/Q(t^*)$ changes from 0.89 to 2.71 for the BC model and from 1.28 to 2.78 for the MBC model. Increase in reversible deformations is accompanied by a decrease in irreversible ones. Thus, at α = 2, δ = 1, El = 10, We = 10 $Z(\infty)/Z_n(\infty)$ = 2.44, 1.28 for BC, and 1.9, 1.0 for MBC; at We = 10, El = 100 $Z(\infty)/Z_n(\infty)$ = 1.42 for BC.

The reverse flow during flow termination first appears near the outer wall, the region involved being larger, the larger δ.

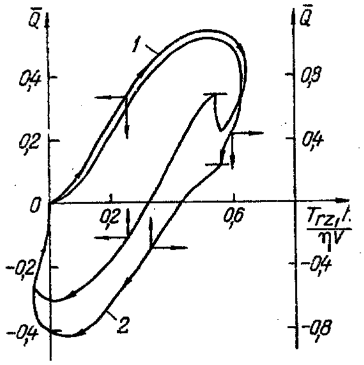

Fig. 2

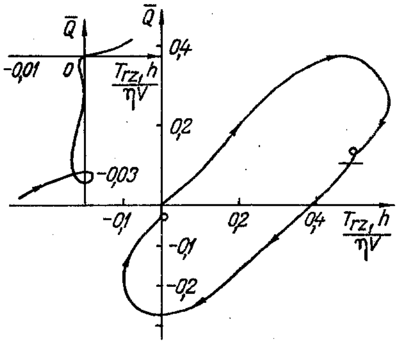

Fig. 3

Fig. 2. $\overline{Q} - T_{rz_1}$ diagram for BC model with $\delta = 1$, $\alpha = 2$, We = 10: El = 10 (1) and El = 100 (2). Line indicates time of pressure removal, arrow indicates direction of time flow.

Fig. 3. Relationship between flow rate $\overline{Q}$ and tangent stress on interior cylinder T_{rz_1} for MBC model at $\delta = 1$, $\alpha = 2$, We = 1, El = 10. Arrow indicates direction of time flow; line, time of pressure removal.

Under nonisothermal conditions, upon application or removal of a pressure gradient, just as in the isothermal analog, reversible deformations decrease, and irreversible ones increase with increase in pressure gradient and decrease in the parameters λ and α. However, a number of unique features do appear, related to the value of the temperature differential and its sign. Nonisothermal conditions level out the quasistationary state upon both application and removal of pressure. In the $\overline{Q} - T_{rz_1}$ diagram zones appear corresponding to significant change in T_{rz_1} with only a slight change in flow rate. In the case of a heated exterior cylinder the stationary T_{rz_1} values are approached from above, while with the interior cylinder heated they are approached from below. The elastic properties manifest themselves somewhat less intensely when the exterior cylinder is heated: $\nu < 0$. Calculations show that for a MBC-model liquid at We = 1, El = 10, $\alpha = 2$, $\delta = 0.1$ the ratio characterizing reversible deformations $Q_{max}(t > t^*)/Q(t^*)$ for $\nu = -0.25$ and $\nu = +0.25$ is 0.94. Irreversible deformations increase: $Z(\infty)|_{\nu=-0.25}/Z(\infty)|_{\nu=0.25} = 1.07$. For isothermal conditions $Q_{max}(t > t^*)/Q(t^*) = 2.77$, while $Z(\infty)/Z_n(\infty) = 1.81$. The total quantity of fluid passing through the tube after application and removal of the pressure gradient in a constant temperature field is less than under isothermal conditions: $Z_-(\infty)/Z_0(\infty) = 0.96$; $Z_+(\infty)/Z_0(\infty) = 0.89$. In the initial period after pressure removal the reverse flow is more intense near the cold wall. Then for either application or removal of pressure the velocity profiles shift toward the heated cylinder.

Using the MBC model, we will consider a change in pressure differential which follows an exponential law:

$$\frac{\partial p}{\partial z}(t) = \begin{cases} \left(\frac{\partial p}{\partial z}\right)_0 [1 - \exp(-t/t_s)], & 0 \leqslant t \leqslant t^*, \\ \frac{\partial p}{\partial z}(t^*) \exp[(t^* - t)/t_s], & t > t^*. \end{cases}$$

Pressure removal occurs in the steady-state flow stage $t^* \gg \lambda$, $t^* \gg \rho h^2/\eta_0$. The following limiting situations are possible: for El >> 1

$$1)\ t_s \ll \rho h^2/\eta_0 \ll \lambda,\quad 2)\ \rho h^2/\eta_0 \ll t_s \ll \lambda,\quad 3)\ \rho h^2/\eta_0 \ll \lambda \ll t_s$$

and for El << 1

$$4)\ t_s \ll \lambda \ll \rho h^2/\eta_0,\ 5)\ \lambda \ll t_s \ll \rho h^2/\eta_0,\ 6)\ \lambda \ll \rho h^2/\eta_0 \ll t_s.$$

In cases 1) and 4) the pressure gradient changes over a time significantly less than the characteristic flow times. Thus, the liquid behaves as though the pressure were removed impulsively. Numerically calculated curves for instantaneous and exponential pressure change under these conditions agree completely. In case 1) the elastic forces play a large role, and the stress and velocity amplitude oscillations are large, while in case 4), when $\rho h^2/\eta_0 >> \lambda$, the liquid behaves as though it is viscous. The same type of flow, close to viscous, is characteristic of case 5). The $\overline{Q} - T_{rz_1}$ diagrams for 4) and 5) are similar: the removal curve lies above the application curve, curling is almost absent, and upon pressure removal there is a "flare" in stress with flow rate remaining practically constant.

For a very slow change in pressure gradient [cases 3) and 6)] the liquid follows the change smoothly. Elastic properties manifest themselves at El >> 1 in the preservation of characteristic inflections in the flow-rate curve in the initial stage of the flow. These are absent at El << 1.

In case 2) over the period of the wave stage the pressure differential is practically constant. Flow rate and tangent stress increase approximately linearly. Then elasticity begins to manifest itself — flow rate and stress oscillations appear, as well as reverse flow in the case of pressure removal.

An experimental study of elastic reversal was performed in [7] for flow of an aqueous solution of carbomethylcellulose in a circular tube. After removal of an impulsively applied pressure gradient, liquid moved in the reverse direction, the displacement reaching 20%. In the calculations performed above, a value of this order of magnitude was obtained at El and We numbers of approximately 10, with $\alpha = 2$.

NOTATION

r, φ, z, cylindrical coordinates; R_1, interior cylinder radius; R_2, exterior cylinder radius; t, time; $C_t(t)$, $C_t^{-1}(t)$, Cauchy and Finger finite deformation tensors; E, unit tensor; D, deformation rate tensor; $m(t)$, memory function; ε, model parameter; α, relaxation time spectrum parameter; $\zeta(\alpha)$, Riemann zeta-function; tr, tensor trace operator; λ_k, relaxation time; λ, maximum relaxation time in spectrum; η_0, initial viscosity; η_k, constants with dimensions of viscosity; ρ, liquid density; $\partial p/\partial z$, pressure gradient; **T**, excess stress tensor; θ, temperature; v_z, z-component of velocity; V, characteristic velocity; E_a, process activation energy; R, universal gas constant; $\overline{Q} = Q/2\pi R_1^2 V\delta$, dimensionless flow rate.

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